MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2016/2017

BNA3014 –NUMERICAL ANALYSIS

(All sections / Groups)

18 OCTOBER 2016
9.00 a.m – 11.00 a.m
(2 Hours)

INSTRUCTIONS TO STUDENT

1. This Question paper consists of **4 pages** including cover page and formulas.
2. Attempt **ALL** questions and write your answers in the Answer Booklet provided.

Question 1 (12.5 marks)

(a) Rewrite the polynomial in **nested form** and evaluate at $x = -1/2$:

$$P(x) = 6x^3 - 2x^2 - 3x + 7$$

[2 marks]

(b) Use the **Intermediate Value Theorem** to prove that $f(x) = 0$ for some $0 < c < 1$.

$$f(x) = x^3 - 4x + 1$$

[2 marks]

(c) Find the root of the equation $f(x) = 2x^3 + 3x^2 - 3x - 5$ by using **Bisection** method with tolerance 0.005. Use $[a, b] = [1, 2]$ as the starting interval.

Note: Write your answer correct to 6 decimal places.

[4.5 marks]

(d) Find the root of the following equation by using **Newton-Raphson** method with tolerance 1×10^{-4}.

$f(x) = 2x^3 + 3x^2 - 3x - 5$ starting with $p_0 = 1$

Note: Write your answer correct to 6 decimal places.

[4 marks]

Question 2 (12.5 marks)

(a) Solve the following set of equations by **Gauss-Jordon elimination** method.

$$\begin{aligned} 9x_1 + 4x_2 + x_3 &= -17 \\ x_1 - 2x_2 - 6x_3 &= 14 \\ x_1 + 6x_2 &= 4 \end{aligned}$$

[6 marks]

(b) Solve the following set of equations by using **Gauss Siedal** iteration with tolerance 1×10^{-4} (start with $x = 0, y = 0, z = 0$).

$$\begin{aligned} 4x + y - z &= 13 \\ x - 5y - z &= -8 \\ 2x - y - 6z &= -2 \end{aligned}$$

Note: Write your answer correct to 7 decimal places.

[6.5 marks]

Continued...

Question 3 (12.5 marks)

(a) Use **Lagrange interpolation** to find a polynomial that passes through the points

i. (0,1), (2,3), (3,0)
ii. (−1,0), (2,1), (3,1), (5,2)

[4 marks]

(b) Consider a function $f(x)$ such that $f(2) = 1.5713$, $f(3) = 1.5719$, $f(5) = 1.5738$, and $f(6) = 1.5751$.

i. Construct a **divided difference** table.
ii. Estimate $f(4)$ using a **second degree** interpolating polynomial, round the answer to 8 decimal places.
iii. Estimate $f(4)$ using a **third degree** interpolating polynomial, round the answer to 8 decimal places.
iv. Is there any **advantage** here in using third degree polynomial?

[8.5 marks]

Question 4 (12.5 marks)

(a) Find the best least square **linear line** through the set of data points and the RMSE:

$$(-3,3), (-1,2), (0,1), (1,-1), (3,-4)$$

[4 marks]

(b) Given the following initial value differential equation

$$y' - y = x, \quad \text{for } 0 \le x \le 1 \text{ with } y(0) = 0 \text{ and } h = 0.2$$

Compute y_1, y_2, and y_3 (5 decimal places) by using each of the following methods

i. **Euler's** Method
ii. **Runge-Kutta second order** method

[8.5 marks]

End of Page

FORMULAS

Taylor Polynomial

$$P_n(x) = \sum_{k=0}^{n} \frac{1}{k!} f^{(k)}(a)(x-a)^k$$

Lagrange Polynomial

$$P_n(x) = \sum_{i=0}^{n} f(x_i) L_i(x)$$

Newton's Polynomial

$$P_n(x) = a_0 + a_1(x - x_0) + \ldots + a_n(x - x_0)(x - x_1)\ldots(x - x_{n-1})$$

The Secant Method

$$p_{n+1} = p_n - \frac{f(p_n)(p_n - p_{n-1})}{f(p_n) - f(p_{n-1})}$$

Newton-Raphson Method

$$p_{n+1} = p_n - \frac{f(P_n)}{f'(P_n)}$$

Euler Method

$$y_{n+1} = y_n + hf(x_n, y_n); \qquad n = 0,1,2,3,\cdots .$$

Runge-Kutta second order methods

$$k_1 = f(x_n, \quad y_n)$$

$$k_2 = f(x_n + h, \quad y_n + hk_1)$$

$$y_{n+1} = y_n + \frac{h}{2}(k_1 + k_2)$$

Simple Linear Regression

$$b_1 = \frac{n\sum_{i=1}^{n} x_i y_i - \left(\sum_{i=1}^{n} x_i\right)\left(\sum_{i=1}^{n} y_i\right)}{n\sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2}$$

$$b_0 = \frac{\sum_{i=1}^{n} y_i - b_1 \sum_{i=1}^{n} x_i}{n} = \bar{y} - b_1 \bar{x}$$